# Linear Equations

Definition: A linear equation in one variable is an equation that can be written in the form $ax+b=c$, where $a, b,$ and $c$ are real numbers and $a \neq 0$

$x+3=-5$ the largest exponent is 1.

$2x-5=4x+1$, $3(x+4)=8$
not in the form $ax+b=c$ but we can rewrite them in this form.

Linear means straight line.
So when we graph a linear equation we get a straight line on the graph.

$x^2+5x-6=0$ and $x^3=8$
are not linear. They have exponents greater than 1 ($>1$)

## Solving a Linear equation

Procedure: Using the Addition Property of Equality to Solve an Equation

Step 1: Determine the operation that will isolate the variable on one side of the equation. Perform this operation on each side of the equation. Remember the inverse property for addition: $a+(-a)=-a+a=a-a=0$

Step 2: Simplify each side of the equation, as necessary. The result should be of the form

$x=$ some number or some number $=x$

Step 3: Check the solution by substituting the value into the original equation.

Step 4: Write the solution in set notation.

Examples:

$$\begin{array}{r} x+12=10 \\ -12 \quad -12 \\ \hline x \quad =-2 \end{array}$$ Subtract 12 from each side

Check
$-2+12=10$
$10=10$

$b + \frac{1}{2} = -\frac{3}{2}$

$-\frac{1}{2} \quad -\frac{1}{2}$

$b = -\frac{4}{2}$

$b = -2$

Check

$-2 + \frac{1}{2} = -\frac{3}{2}$

$-\frac{4}{2} + \frac{1}{2} = -\frac{3}{2}$

$-\frac{3}{2} = -\frac{3}{2}$

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$1.8 = 2.1 + r$

$-2.1 \quad -2.1$

$-0.3 = r$

Check

$1.8 = 2.1 + (-0.3)$

$1.8 = 2.1 - 0.3$

$1.8 = 1.8$

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$4 + x - 3 = -2$

$4 - 3 + x = -2$

$1 + x = -2$

$-1 \quad -1$

$x = -3$

Check

$4 + (-3) - 3 = -2$

$4 - 3 - 3 = -2$

$4 - 6 = -2$

$-2 = -2$

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$6a - 3 = 5a + 1$

$+3 \quad +3$

$6a = 5a + 4$

$-5a \quad -5a$

$a = 4$

Check

$6(4) - 3 = 5(4) + 1$

$24 - 3 = 20 + 1$

$21 = 21$

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$3(x+6) = 4(x-1)$ distribute

$3x + 18 = 4x - 4$

$-3x + 4 \quad -3x + 4$

$22 = x$

Check

$3(22+6) = 4(22-1)$

$3(28) = 4(21)$

$84 = 84$

# Linear Inequalities

**Definition:** A linear inequality in one variable is an inequality of the form $ax+b<c$, where $a$, $b$, and $c$ are real numbers with $a \neq 0$. Note that a linear inequality can have any of the inequality symbols:

$$<,\ \leq,\ >,\ \text{or}\ \geq$$

$$2x+3<-7, \quad 7x-2 \geq 6x+4, \quad 4(3y-1)<-2(y-9)$$

**Procedure: Solving a Linear Inequality**

**Step 1:** Clear any parentheses from the equation by applying the distributive property.

**Step 2:** Remove any fractions by multiplying by the LCD

**Step 3:** Use the addition property of inequality to collect all variable terms on one side and all constant terms on the other side.

**Step 4:** Use the multiplication property of inequality to get a coefficient of 1 on the variable. Remember that multiplying or dividing by a negative number, reverses the inequality symbol.

**Step 5:** If the inequality is a compound inequality of the form $a<x<b$, then the variable must be isolated in the middle. Any operation that is required to isolate the variable must be done to all three parts of the inequality

**Step 6:** Graph the solution set.

**Step 7:** Write the solution set in interval notation or set-builder notation.

**Examples:**

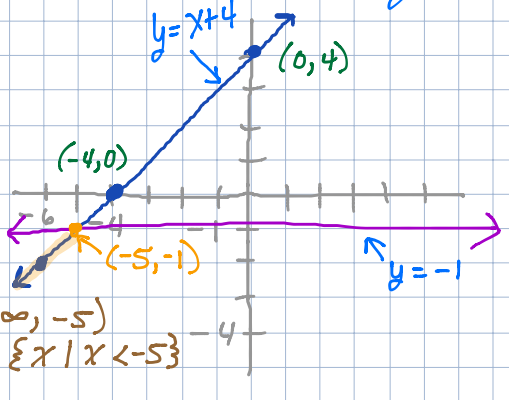
$$x+4<-1$$
$$\frac{-4}{x} \quad \frac{-4}{<-5}$$


$y=x+4$

| $x$ | $y$ |
|---|---|
| 0 | 4 |
| -4 | 0 |

$y=0+4$
$y=4$

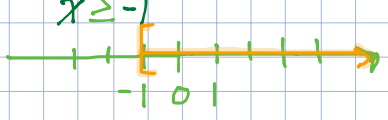
$0=x+4$
$-4 \quad -4$
$-4=x$




Interval notation $(-\infty, -5)$
Set-builder notation $\{x \mid x<-5\}$

$\frac{-4x}{-4} \leq \frac{4}{-4}$

$x \geq -1$




Interval notation $[-1, \infty)$

Set-builder notation $\{x \mid x \geq -1\}$

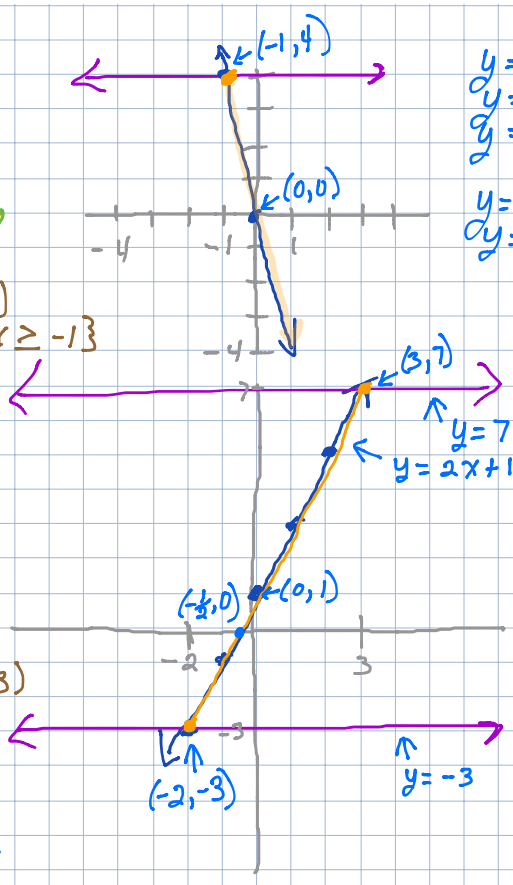
$-3 < 2x+1 < 7$

$\frac{-1}{-4} < \frac{2x}{2} < \frac{6}{2}$

$-2 < x < 3$


Interval Notation $(-2, 3)$

Set-builder Notation $\{x \mid -2 < x < 3\}$




$y = -4x$

$y = -4(0)$

$y = 0$

$y = -4(-1)$

$y = 4$

| x | y |
|---|---|
| 0 | 0 |
| -1 | 4 |

$y = 2x+1$

$y = 2(0)+1$

$y = 1$

| x | y |
|---|---|
| 0 | 1 |
| $-\frac{1}{2}$ | 0 |
| -2 | 3 |
| 3 | 7 |

$0 = 2x+1$

$\frac{-1}{2} = \frac{-1}{2}$

$\frac{-1}{2} = \frac{2x}{2}$

$-\frac{1}{2} = x$

from this solution
Substitute -2 for x
then 3 for x
$2(-2)+1 = -3$
$2(3)+1 = 7$
put these points in your table